Optimal Observer-based Output Feedback Controller for Traffic Congestion with Bottleneck

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Outline

1. Context
2. Contributions
3. Content
   - Traffic flow system and control problem
   - Full-state feedback controller
   - Observer design and Output feedback controller
   - Optimal controller and numerical studies
4. Ongoing work

Context

- Traffic congestion resulting from traffic breakdown: increase of fuel consumption and unsafe driving conditions
- Three factors of traffic breakdown: high traffic demand, bottlenecks, and disturbances caused by individual drivers [Treiber M,2012]
- Macroscopic traffic models: first order LWR model [Lighthill MJ,1955][Richards PI,1956], second order PW model [Payne HJ,1971][Whitham GB,1999], and second order ARZ model[Aw A,2000][Zhang HM,2002]
- Design a boundary control law: spectral analysis[Belletti F,2015], Lyapunov method[Zhang L,2019], backstepping method
- Backstepping: initially introduced for hyperbolic PDEs by [Krstic M,2008][Smyshlyaev A,2010][Coron JM,2013]; theoretical results [Deutscher J,2017][Anfinsen H,2018][Yu H,2020][Auriol J,2020][Vazquez R,2011][Yu H,2019][Anfinsen H,2019]

Contributions

- First work to design an optimal observer-based output boundary control of traffic breakdown to remove or weaken the effect of high traffic demand acting as a time-varying disturbance input with the fastest convergence rate;
- Design a controller and an observer respectively with a time-depending integral term to reject disturbances;
- Use Lyapunov approach to prove the iISS of PDEs target system with a PI boundary control strategy;
- Inspired by [Anfinsen H,2019], compute the kernels of backstepping transformations using a general expression of the kernel functions.

ARZ traffic flow model $(\rho, v)^\top$

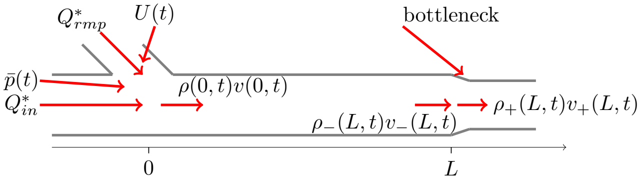
$$\rho_t + (v\rho)_x = 0,$$
$$v_t + \left(v - \rho p'(\rho)\right) v_x = \frac{V_e(\rho) - v}{\tau},$$

- $\rho(x,t)$ – traffic density; $v(x,t)$ – mean speed;
- x – independent space variable in $(0, L)$ on a road section of length L; t – independent time variable in $[0, \infty)$;
- τ – speed adaptation time, corresponding to inverse of agility;
- $V_e(\rho)$ – steady-state speed, given by Greenshields model[Greenshields,1935],

  $$V_e(\rho) = v_f \left(1 - \frac{\rho}{\rho_m}\right),$$

  v_f – free flow speed, ρ_m – maximum density;
- $p(\rho)$ – traffic pressure, $p(\rho) = v_f - V_e(\rho) = \frac{v_f}{\rho_m}\rho$.

To increase efficiency and stability, we solve optimal control problem of high traffic demand by ramp metering with a bottleneck and disturbances.



- Constant density $\rho_-(L,t) = \rho_+(L,t) = \rho^*_{out}$ and speed drop $v_-(L,t) > v_+(L,t)$;
- $Q^*_{in} + Q^*_{rmp} + U(t) + \bar{p}(t) = \rho(0,t)v(0,t)$;
- Q^*_{rmp} – steady-state on-ramp flow;
- Q^*_{in} – steady-state inflow, $Q^*_{in} + Q^*_{rmp} = \rho^*(0)v^*(0)$;
- $\bar{p}(t)$ – unknown disturbances of inflow;
- Model output: $y(t) = v(0,t) - v^*(0)$;
- $U(t)$ – control law implemented by on-ramp metering, optimal gains.

Quasilinear deviation system $(\widetilde{\omega}, \widetilde{v})^\top$

$$\widetilde{\omega}_t + \Lambda_1(\widetilde{\omega}, \widetilde{v}, x)\widetilde{\omega}_x + \widetilde{v}\frac{\mathrm{d}\omega^*(x)}{\mathrm{d}x} = -\frac{\widetilde{\omega}}{\tau}$$

$$\widetilde{v}_t + \Lambda_2(\widetilde{\omega}, \widetilde{v}, x)\widetilde{v}_x + (2\widetilde{v} - \widetilde{\omega})\frac{\mathrm{d}v^*(x)}{\mathrm{d}x} = -\frac{\widetilde{\omega}}{\tau}$$

with $\widetilde{\omega} = \omega - \omega^*(x)$, $\widetilde{v} = v - v^*(x)$ and $\omega = v + \frac{v_f}{\rho_m}\rho$;

- characteristic velocities

$$\Lambda_1(\widetilde{\omega}, \widetilde{v}, x) = \widetilde{v} + v^*(x) > 0$$
$$\Lambda_2(\widetilde{\omega}, \widetilde{v}, x) = 2\widetilde{v} - \widetilde{\omega} + 2v^*(x) - \omega^*(x) < 0$$

- steady state $(\omega^*(x), v^*(x))^\top \in \mathcal{C}^1([0, L]; \mathbb{R}^2)$ such that $2v^*(x) - \omega^*(x) < 0$ for all $x \in [0, L]$.

Linearized system $(\varepsilon_1, \varepsilon_2)^\top$

$$\begin{aligned}
&\varepsilon_{1t} + \lambda_1(x)\varepsilon_{1x} + c_1(x)\varepsilon_2 = 0,\\
&\varepsilon_{2t} - \lambda_2(x)\varepsilon_{2x} + c_2(x)\varepsilon_1 = 0,\\
&\varepsilon_1(0,t) = q_1\varepsilon_2(0,t) + q_2\left(U(t) + \bar{p}(t)\right),\\
&\varepsilon_2(L,t) = \frac{1}{\kappa}\varepsilon_1(L,t),
\end{aligned}$$

with coordinate transformation $\varepsilon_1 = \psi_1(x)\widetilde{\omega}, \varepsilon_2 = \psi_2(x)\widetilde{v}$.

- $\lambda_1(x) > 0$, $\lambda_2(x) > 0$;
- known parameters q_1, q_2, and κ.

Content – Full-state feedback controller

Target system

$$\alpha_t(x,t) + \lambda_1(x)\alpha_x(x,t) = 0,$$
$$\beta_t(x,t) - \lambda_2(x)\beta_x(x,t) = 0,$$
$$\alpha(0,t) = q_1\beta(0,t) + k_i\eta(t),$$
$$\beta(L,t) = \frac{1}{\kappa}\alpha(L,t),$$
$$\eta(t) = \int_0^t (\beta(0,s) - \alpha(0,s))\,\mathrm{d}s + k_i^{-1} q_2 \bar{p}(t),$$

- $k_i \in \mathbb{R} \setminus \{0\}$ – integral tuning parameter.

Theorem 1. Integral Input-to-state Stability of Target System

Assume there exist positive constants μ, θ, p_1, p_2, p_4, q_3, q_4 and constant p_3 such that for all x in $[0, L]$,

$$M_1(x) \geq 0.$$

Then there exists positive constants Ω_1, b_1 such that, for any $z_0 = (\alpha(\cdot, 0), \beta(\cdot, 0), \eta(0))^\top$ in $L^2((0, L); \mathbb{R}^3)$, and for any $\bar{p}$ such that $\dot{\bar{p}} \in \mathcal{L}^2[0, \infty)$, the solution $z = (\alpha, \beta, \eta)^\top$ to the target system satisfies, for all $t \geq 0$,

$$\|z(\cdot, t)\|^2_{L^2((0,L);\mathbb{R}^3)} \leq \Omega_1 e^{-\theta t} \|z_0\|^2_{L^2((0,L);\mathbb{R}^3)} + b_1 \int_0^t \dot{\bar{p}}^2(s)\,\mathrm{d}s.$$

1st backstepping transformation[Coron JM,2013]: linearized $(\varepsilon_1, \varepsilon_2)^\top$ – target $(\alpha, \beta)^\top$

$$\alpha = \varepsilon_1 - \int_x^L G^{11}(x,\xi)\varepsilon_1(\xi,t)\,\mathrm{d}\xi - \int_x^L G^{12}(x,\xi)\varepsilon_2(\xi,t)\,\mathrm{d}\xi,$$
$$\beta = \varepsilon_2 - \int_x^L G^{21}(x,\xi)\varepsilon_1(\xi,t)\,\mathrm{d}\xi - \int_x^L G^{22}(x,\xi)\varepsilon_2(\xi,t)\,\mathrm{d}\xi,$$

- kernels $G^{11}(x,\xi)$, $G^{12}(x,\xi)$, $G^{21}(x,\xi)$ and $G^{22}(x,\xi)$ in the triangular domain $\mathbb{T}_1 = \big\{(x,\xi) \in \mathbb{R}^2 \,|\, 0 \le x \le \xi \le L\big\}$;
- full-state feedback control law $U(t)$;
- invertibility of backstepping transformation, linearized system is iISS.

Observer $(\hat{\varepsilon}_1, \hat{\varepsilon}_2)^\top$ design for estimation of state $(\varepsilon_1, \varepsilon_2)^\top$

$$\hat{\varepsilon}_{1t} + \lambda_1(x)\hat{\varepsilon}_{1x} + c_1(x)\hat{\varepsilon}_2 = r(x)\left(y(t) - \hat{\varepsilon}_2(0,t)\right)$$
$$\hat{\varepsilon}_{2t} - \lambda_2(x)\hat{\varepsilon}_{2x} + c_2(x)\hat{\varepsilon}_1 = s(x)\left(y(t) - \hat{\varepsilon}_2(0,t)\right)$$
$$\hat{\varepsilon}_1(0,t) = q_1\hat{\varepsilon}_2(0,t) - L_i \int_0^t \left(y(\tau) - \hat{\varepsilon}_2(0,\tau)\right) \mathrm{d}\tau + q_2 U(t)$$
$$\hat{\varepsilon}_2(L,t) = \frac{1}{\kappa}\hat{\varepsilon}_1(L,t)$$

- $r(x)$, $s(x)$ – output injection gains
- $L_i \in \mathbb{R} \setminus \{0\}$ – integral tuning parameter
- Integral term is added to reject perturbation to guarantee the convergence of estimated state to real state

Error system $(\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_2)^\top$

$$
\begin{aligned}
&\widetilde{\varepsilon}_{1t} + \lambda_1(x)\widetilde{\varepsilon}_{1x} + c_1(x)\widetilde{\varepsilon}_2 = -r(x)\widetilde{\varepsilon}_2(0,t) \\
&\widetilde{\varepsilon}_{2t} - \lambda_2(x)\widetilde{\varepsilon}_{2x} + c_2(x)\widetilde{\varepsilon}_1 = -s(x)\widetilde{\varepsilon}_2(0,t) \\
&\widetilde{\varepsilon}_1(0,t) = q_1\widetilde{\varepsilon}_2(0,t) + L_i \int_0^t \widetilde{\varepsilon}_2(0,\tau)\,\mathrm{d}\tau + q_2\bar{p}(t) \\
&\widetilde{\varepsilon}_2(L,t) = \frac{1}{\kappa}\widetilde{\varepsilon}_1(L,t) \\
&\widetilde{\eta}(t) = \int_0^t \widetilde{\varepsilon}_2(0,\tau)\,\mathrm{d}\tau + L_i^{-1} q_2 \bar{p}(t)
\end{aligned}
$$

with $\widetilde{\varepsilon}_1 = \varepsilon_1 - \hat{\varepsilon}_1$, $\widetilde{\varepsilon}_2 = \varepsilon_2 - \hat{\varepsilon}_2$

2nd backstepping transformation: target $(\alpha, \beta)^\top$ – error $(\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_2)^\top$

$$\widetilde{\varepsilon}_1 = \alpha + \int_0^x F^{11}(x,\xi)\alpha(\xi,t)\,\mathrm{d}\xi + \int_0^x F^{12}(x,\xi)\beta(\xi,t)\,\mathrm{d}\xi$$
$$\widetilde{\varepsilon}_2 = \beta + \int_0^x F^{21}(x,\xi)\alpha(\xi,t)\,\mathrm{d}\xi + \int_0^x F^{22}(x,\xi)\beta(\xi,t)\,\mathrm{d}\xi$$

- kernels $F^{ij}(x,\xi)$ in $L^2((0,L)^2;\mathbb{R})$, $i,j = 1,2$ in the triangular domain $\mathbb{T} = \big\{(x,\xi) \in \mathbb{R}^2 \,|\, 0 \leq \xi \leq x \leq L\big\}$;
- injection gains are, for all x in $[0, L]$,

$$r(x) = \lambda_2(0)F^{12}(x,0) - \left(1 - \frac{L_i}{k_i}\right)\lambda_1(0)F^{11}(x,0),$$
$$s(x) = \lambda_2(0)F^{22}(x,0) - \left(1 - \frac{L_i}{k_i}\right)\lambda_1(0)F^{21}(x,0).$$

Theorem 2 *iISS* of Error System

Under the assumptions of Theorem 1, consider the error system and the functions r and s, the equilibrium $\widetilde{\varepsilon}_1 \equiv \widetilde{\varepsilon}_2 \equiv 0$ is iISS in the L^2 sense, that is there exists positive constants Ω_2, b_2 such that, for any $(\widetilde{\varepsilon}_1(\cdot,0), \widetilde{\varepsilon}_2(\cdot,0))^\top$ in $L^2((0,L);\mathbb{R}^2)$, and for any $\bar{p}$ such that $\dot{\bar{p}} \in \mathcal{L}^2[0,\infty)$, the solution $(\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_2, \widetilde{\eta})^\top$ to the error system satisfies

$$\int_0^L \left(\widetilde{\varepsilon}_1^2 + \widetilde{\varepsilon}_2^2\right) \,\mathrm{d}x + L\widetilde{\eta}^2 \leq \Omega_2 e^{-\theta t}\Big[\int_0^L (\widetilde{\varepsilon}_1^2(x,0) + \widetilde{\varepsilon}_2^2(x,0))\,\mathrm{d}x + L\widetilde{\eta}^2(0)\Big] + b_2 \int_0^t \dot{\bar{p}}^2(s)\,\mathrm{d}s,\ t \geq 0.$$

- Observer-based output feedback controller

$$
\begin{aligned}
U(t) = {} & \frac{k_i}{q_2}\int_0^t \left(y(s) - \hat{\varepsilon}_1(0,s)\right) \mathrm{d}s \\
& + \frac{k_i}{q_2}\int_0^t\int_0^L \Big[\left(G^{11}(0,\xi) - G^{21}(0,\xi)\right) \hat{\varepsilon}_1(\xi,s) \\
& \qquad\qquad + \left(G^{12}(0,\xi) - G^{22}(0,\xi)\right) \hat{\varepsilon}_2(\xi,s) \Big] \mathrm{d}\xi \, \mathrm{d}s \\
& + \frac{1}{q_2}\int_0^L \Bigg[\left(G^{11}(0,\xi) - q_1 G^{21}(0,\xi)\right) \hat{\varepsilon}_1(\xi,t) \\
& \qquad\qquad + \left(G^{12}(0,\xi) - q_1 G^{22}(0,\xi)\right) \hat{\varepsilon}_2(\xi,t) \Bigg] \mathrm{d}\xi
\end{aligned}
$$

Theorem 3 iISS of Closed-loop System

Under the assumptions of Theorem 1, for any $(\varepsilon_1(\cdot,0),\varepsilon_2(\cdot,0),\widetilde{\varepsilon}_1(\cdot,0),\widetilde{\varepsilon}_2(\cdot,0))^\top$ in $L^2((0,L);\mathbb{R}^4)$, the observer-based output feedback controller makes the equilibrium of the linearized system and the error system iISS in the L^2 sense, that is there exists positive constants Ω_3, b_3 such that along the solution to linearized system, for any $\bar{p}$ such that $\dot{\bar{p}} \in \mathcal{L}^2[0,\infty)$, it holds, for all t in $[0,\infty)$,

$$
\begin{aligned}
&\int_0^L \left(\varepsilon_1^2+\varepsilon_2^2\right)\,\mathrm{d}x + L\eta^2(t) \\
&\qquad + \int_0^L \left(\widetilde{\varepsilon}_1^2+\widetilde{\varepsilon}_2^2\right)\,\mathrm{d}x + L\widetilde{\eta}^2(t) \\
&\le \Omega_3 e^{-\theta t}\Big[\int_0^L \left(\varepsilon_1^2(x,0)+\varepsilon_2^2(x,0)\right)\,\mathrm{d}x + L\eta^2(0) \\
&\qquad + \int_0^L \left(\widetilde{\varepsilon}_1^2(x,0)+\widetilde{\varepsilon}_2^2(x,0)\right)\,\mathrm{d}x + L\widetilde{\eta}^2(0)\Big] + b_3\int_0^t \dot{\bar{p}}^2(s)\,\mathrm{d}s.
\end{aligned}
$$

- Optimization problem: maximal θ can be considered to derive the optimal value of k_i (optimal control law $U(t)$)

  max θ

  subject to $\mu, \theta, p_1, p_2, p_4, q_3, q_4 > 0$, and $M_1(x) \geq 0$ for all $x \in [0, L]$.

- Simulations: Given $\bar{p}(t), \omega^*(x), v^*(x)$, compute controller for the linearized system and simulate the nonlinear model.

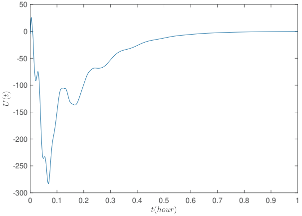


Figure: Evolution of the observer-based output feedback controller $U(t)$.

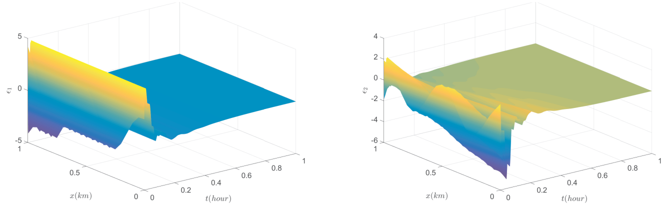


Figure: State $(\varepsilon_1, \varepsilon_2)^\top$ of the closed-loop system.

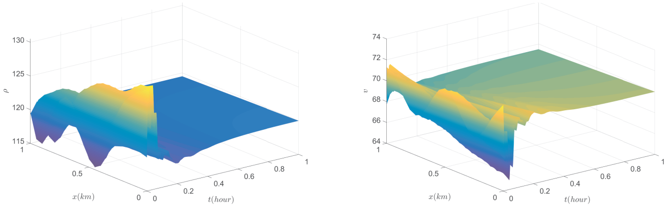


Figure: State $(\rho, v)^\top$ of the plant system.

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