Control of linear and nonlinear systems subject to a random input delay

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joint work with SiJia Kong

Webinar Control in Time of Crisis
May 27, 2021

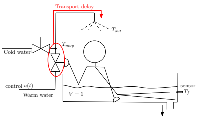
MINES ParisTech

Time-Varying Delays are not necessarily FIFO...

- **Transport delays**

"First-in/First-out" principle
transportation of material

- Bath/shower



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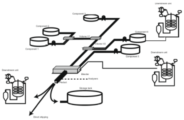

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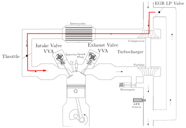

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$$\boxed{\dot{D}(t) < 1\,,\, t \geq 0}$$

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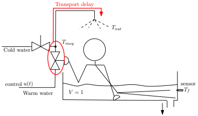

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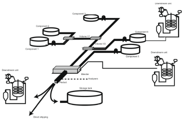

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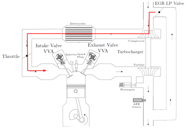

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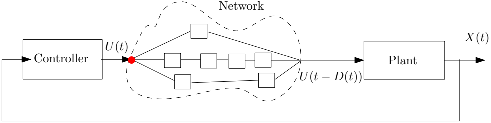

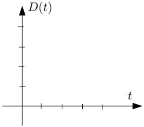

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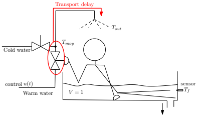

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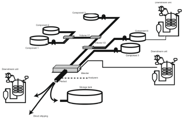

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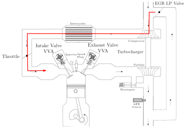

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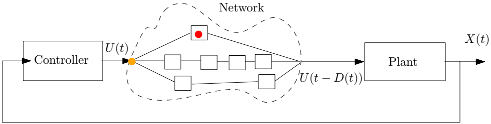

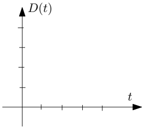

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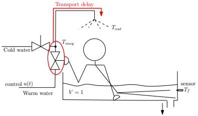

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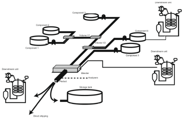

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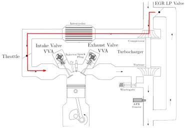

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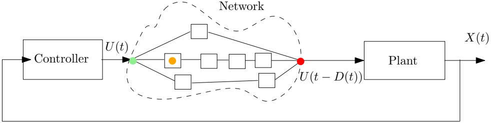

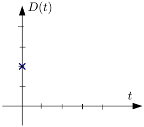

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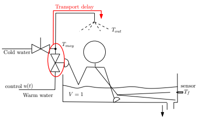

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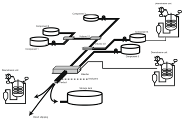

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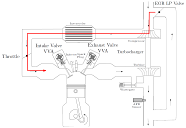

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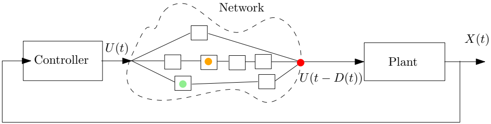

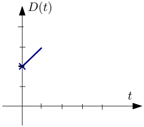

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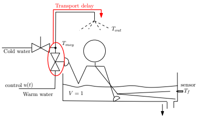

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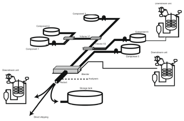

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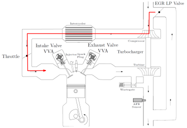

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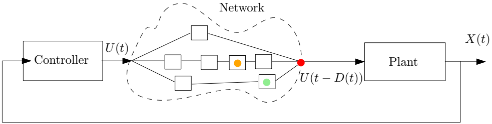

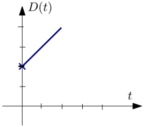

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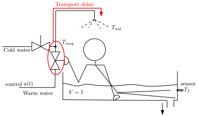

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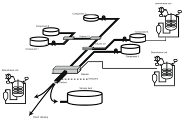

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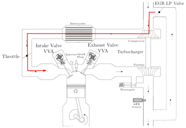

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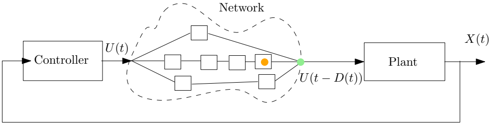

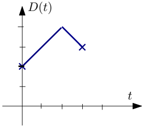

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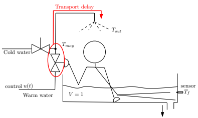

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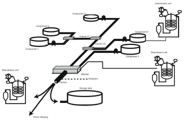

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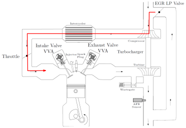

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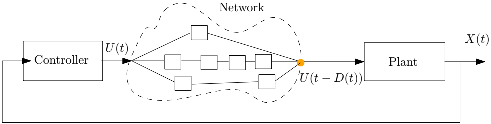

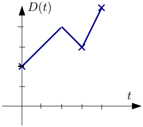

sudden variation : random delay

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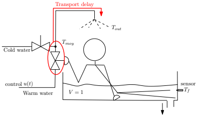

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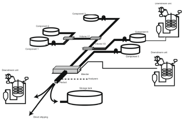

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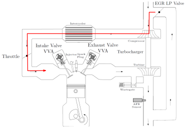
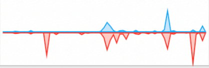


- **Communication delay**

Paquets entrants :	14 345 872
Paquets sortants :	16 188 961
Entrée paquets/s :	3
Sortie paquets/s :	3

PAQUETS



Données reçues :	8,73 Go
Données envoyées :	5,30 Go
Données reçues/s :	612 octets
Données envoyées/s :	631 octets

sudden variation : random delay

...but non-FIFO have seldom been studied...

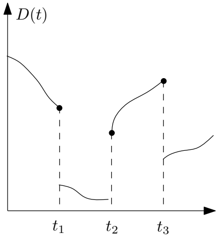
Time-varying delays are almost always assumed FIFO :

- either as a prerequisite of the analysis
- or as a consequence of the control design

except *fast-varying delays* and *sampled-data systems* (Fridman et al.)

Recent works have focused on piecewise differentiable delays :

- F. Mazenc, M. Malisoff, and S.-I. Niculescu. *Stability and control design for time-varying systems with time-varying delays using a trajectory-based approach*, SIAM Journal on Control and Optimization, 2017
- D. Bresch-Pietri, F. Mazenc and N. Petit, *Robust compensation of a chattering time-varying input delay with jumps*, in Automatica, 2018
- J. Choi and M. Krstic, *Compensation of time-varying input delay for discrete-time nonlinear systems*, IJRNC, 2016.



...and neither have stochastic delays.

Most of the works on Stochastic Differential Delay Equations do not consider a random delay

$$dx(t) = (Ax(t) + Bu(t - D(t)))dt + FdW_t$$

Only a few works consider the delay to be a random process in a control perspective :

- piecewise constant process
  H. T. Sykora, M. Sadeghpour, J. I. Ge, D. Bachrathy, and G. Orosz. *On the moment dynamics of stochastically delayed linear control systems*, IJRNC, 2020.
- affine term multiplied by a random boolean
  K. Li and X. Mu. *Predictor-based H∞ leader-following consensus of stochastic multi-agent systems with random input delay*, Optimal Control Applications and Methods, 2021
- Markov process with a finite number of values : constant delay averaging
  I. Kolmanovsky and T. Maizenberg, *Mean-square stability of nonlinear systems with time-varying, random delay*. Stochastic analysis and Applications, 2001.

Problem statement

We consider the linear time-invariant plant

$$\dot{X}(t) = AX(t) + BU(t - D(t))$$

in which X an $\mathbb{R}^n$-valued random variable and $U \in \mathbb{R}$.

Delay definition

D is a Markov process with the following properties :

(1) $D(t) \in \{D_1, D_2, \ldots, D_r\}$, with $0 < \underline{D} \leq D_1 < D_2 < \ldots < D_r \leq \overline{D}$.

(2) The transition probabilities $P_{ij}(t_1, t_2)$, which quantify the probability to switch from D_i at time t_1 to D_j at time t_2 ($(i,j) \in \{1, \ldots, r\}^2$, $t_2 \geq t_1 \geq 0$), are differentiable functions $P_{ij} : \mathbb{R}^2 \to [0, 1]$ satisfying

$$\sum_{j=1}^{r} P_{ij}(t_1, t_2) = 1, \quad (0 \leq t_1 \leq t_2)$$

Input delay compensation

Dynamics at stake

$$\dot{X}(t) = AX(t) + BU(t-D)$$

with $X \in \mathbb{R}^n$, U scalar and $D > 0$ **constant**.
$\exists K \in \mathbb{R}^{1\times n} A + BK$ Hurwitz.

Prediction-based control (Smith,1959, FSA Manitius and Olbrot, 1979)

$$U(t) = KX_P(t+D) = K\left[e^{AD}X(t) + \int_{t-D}^{t} e^{A(t-s)}BU(s)ds\right]$$

yields an exact compensation of the input delay

Closed-loop form

$\dot{X} = (A+BK)X(t)$

$\Rightarrow$ delay-free exponential convergence after D units of time

Compensation of a time-varying delay

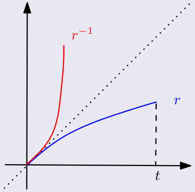
$$\dot{X}(t) = AX + BU(r(t)) \quad \text{with} \quad r(t) = t - D(t)$$

Time-varying prediction horizon

Exact compensation is obtained with

$$U(t) = KX(r^{-1}(t))$$

if
1. D is continuously differentiable and bounded
2. r is invertible (i.e. $|\dot{D}| < 1$)



M. Nihtila. Finite pole assignment for systems with time-varying input delays. *CDC*, 1991

Implementability ?

- $r^{-1}(t) \neq t + D(t) \quad \Rightarrow$ **need to predict future values of the delay**
- Non-smooth delay in the stochastic case !

Robust delay compensation

Numerous delay-robustness properties have been obtained in the deterministic delay case for prediction-based control :

- N. Bekiaris-Liberis and M. Krstic, *Robustness of nonlinear predictor feedback laws to time-and state-dependent delay perturbations*, Automatica, 2013
- I. Karafyllis and M. Krstic, *Delay-robustness of linear predictor feedback without restriction on delay rate. Automatica*, 2013.
- S. Kong and D. Bresch-Pietri, *Constant time horizon prediction-based control for linear systems with time-varying input delay*, in Proc. of the 2020 IFAC World Congress

$\Rightarrow$ We propose to follow this trend in the stochastic delay context.

Outline

1. Robust compensation for linear systems
2. PDE transformations and Lyapunov analysis
3. Simulation results
4. Extension to nonlinear dynamics

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Prediction-based controller

Constant horizon prediction

$$U(t) = K\left[e^{AD_0}X(t) + \int_{t-D_0}^{t} e^{A(t-s)}BU(s)ds\right], \quad t \geq 0$$

in which K is a feedback gain such that $A+BK$ is Hurwitz, and $D_0 \in [\underline{D}, \overline{D}]$.

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Theorem

There exists a positive constant $\varepsilon^\star(K)$ such that, if

$$|D_0 - D_j| \leq \varepsilon^\star(K), \quad j \in \{1, ..., r\}$$

there exist positive constants R and γ such that

$$\mathbb{E}_{[0,\Upsilon(0)]}[\Upsilon(t)] \leq R\Upsilon(0)e^{-\gamma t}$$

with

$$\Upsilon(t) = |X(t)|^2 + \int_{t-\overline{D}-D_0}^{t} U(s)^2 ds$$

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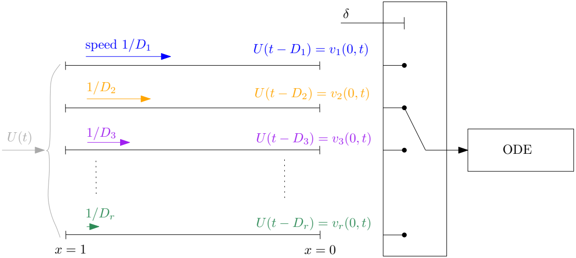
- Transport PDE representation for each delay value

$$v_j(x,t) = U(t + D_j(x-1))$$



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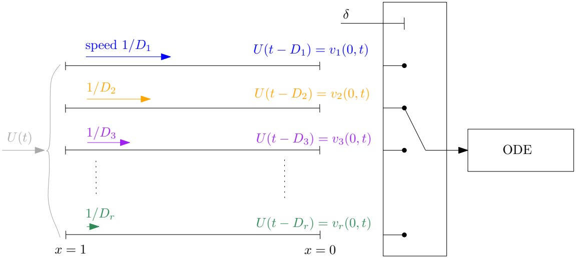

With $\Lambda_D = diag(D_1, ..., D_r)$ and the random process $\delta(t) = e_i$ iff $D(t) = D_i$,

$$\begin{cases} \dot{X}(t) = AX(t) + B\delta(t)^T \mathbf{v}(0,t) \\ \Lambda_D \mathbf{v}_t(x,t) = \mathbf{v}_x(x,t) \\ \mathbf{v}(1,t) = \mathbf{1}U(t) \end{cases}$$

Extended system and backstepping transformation

- Extended system with $\hat{v}(x,t) = U(t + D_0(x-1))$ and $\tilde{\mathbf{v}} = \mathbf{v} - \hat{v}\mathbf{1}$.

$$\begin{cases} \dot{X}(t) = AX(t) + B\hat{v}(0,t) + B\delta(t)^T\tilde{\mathbf{v}}(0,t) \\ D_0\hat{v}_t(x,t) = \hat{v}_x(x,t) \\ \hat{v}(1,t) = U(t) \\ \Lambda_D\tilde{\mathbf{v}}_t(x,t) = \tilde{\mathbf{v}}_x - \Sigma_D\hat{v}_x \\ \tilde{\mathbf{v}}(1,t) = \mathbf{0} \end{cases}$$

in which $\Sigma_D = \left(\frac{D_1 - D_0}{D_0}, \ldots, \frac{D_r - D_0}{D_0}\right)^T$

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- Backstepping transformation $(X, \hat{v}) \mapsto (X, w)$ s.t. $w(1,t) = 0$

$$w(x,t) = \hat{v}(x,t) - Ke^{AD_0x}X(t) - D_0\int_0^x Ke^{AD_0(x-y)}B\hat{v}(y,t)dy$$

Extended system and backstepping transformation

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- Backstepping transformation $(X, \hat{v}) \mapsto (X, w)$ s.t. $w(1,t) = 0$

$$\begin{cases} \dot{X}(t) = (A+BK)X(t) + Bw(0,t) + \delta(t)^T\tilde{\mathbf{v}}(0,t) \\ D_0 w_t(x,t) = w_x(x,t) - D_0 K e^{AD_0 x} B\delta(t)^T\tilde{\mathbf{v}}(0,t) \\ w(1,t) = 0 \\ \Lambda_D\tilde{\mathbf{v}}_t(x,t) = \tilde{\mathbf{v}}_x - \Sigma_D h(w_x, w, X) \\ \tilde{\mathbf{v}}(1,t) = \mathbf{0} \end{cases}$$

Extended system and backstepping transformation

- Final extended target system with $\mu(x,t) = U\left(t - D_0 + \overline{D}(x-1)\right)$

$$\begin{cases} \dot{X}(t) = (A+BK)X(t) + Bw(0,t) + B\delta(t)^T\tilde{\mathbf{v}}(0,t) \\ D_0 w_t(x,t) = w_x(x,t) - D_0 K e^{AD_0 x} B\delta(t)^T \tilde{\mathbf{v}}(0,t) \\ w(1,t) = 0 \\ \Lambda_D \tilde{\mathbf{v}}_t(x,t) = \tilde{\mathbf{v}}_x - \Sigma_D h(\mu, w, X) \\ \tilde{\mathbf{v}}(1,t) = \mathbf{0} \\ \overline{D}\mu_t(x,t) = \mu_x(x,t) \\ \mu(1,t) = KX(t) + w(0,t) \end{cases}$$

in which $\Sigma_D = \left(\frac{D_1 - D_0}{D_0}, \ldots, \frac{D_r - D_0}{D_0}\right)^T$

- Define its state as $\Psi = (X, w, \tilde{\mathbf{v}}, \mu)$ belonging to

$$\mathbb{R}^n \times \mathcal{L}_2([0,1],\mathbb{R}) \times \mathcal{L}_2([0,1],\mathbb{R}^r) \times \mathcal{L}_2([0,1],\mathbb{R}) \triangleq \mathcal{D}_\Psi$$

The closed-loop and the target systems are well-posed

Weak solution

By a weak solution to the closed-loop system, we refer to a $\mathbb{R}^n \times \mathcal{L}_2([-\overline{D},0],\mathbb{R}) \times \mathbb{R}$-valued random variable $(X(X_0,t), U_t(U_0,\cdot), D(t))$, the realizations of which satisfy an integral form of the closed-loop dynamics.

Lemma

For every initial condition $(X_0, U_0) \in \mathbb{R}^n \times \mathcal{L}_2([-\overline{D},0],\mathbb{R})$, the closed-loop system has a unique weak solution defined as

$$X(t) = e^{At}X(0) + \int_0^t e^{A(t-s)}BU(s-D(s))ds$$

Consequently, for each initial condition in $\mathcal{D}_\Psi$, the target system also has a unique weak solution Ψ.

(Ψ, δ) thus defines a continuous-time Markov process.

Stability analysis

Lyapunov functional candidate

$$V(\Psi) = X^T P X + bD_0 \int_0^1 (1+x) w(x)^2 dx + c \sum_{l=1}^{r} \int_0^1 (1+x) \left((e_l \cdot \mathbf{D})^T \tilde{\mathbf{v}}(x) \right)^2 dx$$

$$+ d\overline{D} \int_0^1 (1+x) \mu(x)^2 dx \quad \text{with } b, c, d > 0 \text{ and } P \text{ sol. to a Lyapunov eq.}$$

Infinitesimal Operator

$$LV(\Psi(t)) = \limsup_{\Delta t \to 0^+} \frac{1}{\Delta t} \left(\mathbb{E}_{[t, \Psi(t)]} [V(\Psi(t+\Delta t))] - V(\Psi(t)) \right)$$

Constant delay averaging (Kolmanovsky) :

$$LV(t) = \sum_{j=1}^{r} P_{ij}(0,t) \frac{dV}{d\Psi}(\Psi(t)) \underbrace{f_j(\Psi(t))}_{\text{dynamics for } \delta(t)=\delta_j} + \underbrace{\sum_{j=1}^{r} \frac{\partial P_{ij}}{\partial t}(0,t)}_{= \frac{\partial}{\partial t} \sum_{j=1}^{r} P_{ij}(0,t) = 0} V(t)$$

Stability analysis

Lemma

Assume there exist a positive constant ε such that

$$|D_0 - D_j| \leq \varepsilon, \quad j \in \{1, ..., r\}$$

Then, there exist $b, c, d \in {\mathbb{R}_+^*}^3$ which are independent of ε such that the Lyapunov functional V satisfies

$$LV(t) \leq -(\eta - g(\varepsilon))V(t), \quad t \geq \overline{D}$$

with the function $g : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying $\lim_{\varepsilon \to 0} g(\varepsilon) = 0$.

Stability analysis

- As $\lim_{\varepsilon \to 0} g(\varepsilon) = 0$, there exists $\varepsilon^* > 0$ such that $\eta - g(\varepsilon) = \eta_0 > 0$ for $\varepsilon < \varepsilon^*$. and hence
$$LV(t) \leq -\eta_0 V(t), \quad t \geq \overline{D}$$
- According to Dynkin's formula, one obtains for $\varepsilon < \varepsilon^*$
$$\mathbb{E}_{(0,\Psi(0))}[V(t)] - \mathbb{E}_{(0,\Psi(0))}[V(\overline{D})] \leq \mathbb{E}_{(0,\Psi(0))}\left[\int_{\overline{D}}^{t} -\eta_0 V(s) ds\right]$$
And, applying Gronwall's inequality,
$$\mathbb{E}_{(0,\Psi(0))}[V(t)] \leq \mathbb{E}_{(0,\Psi(0))}[V(\overline{D})] e^{-\eta_0 (t-\overline{D})}$$
- We conclude by using the fact that the system does not escape in finite time and that V and Υ are equivalent.

Outline

1 Robust compensation for linear systems

2 PDE transformations and Lyapunov analysis

3 Simulation results

4 Extension to nonlinear dynamics

Toy example

Dynamics under consideration

$$\dot{X}(t) = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} U(t - D(t))$$

$$U(t) = K \left[e^{AD_0} X(t) + \int_{t-D_0}^{t} e^{A(t-s)} BU(s) ds \right], \quad t \geq 0$$

with the feedback gain $K = -\begin{bmatrix} 1 & 2 \end{bmatrix}$ s.t. $\lambda(A+BK) = -0.5000 \pm 1.3229i$. The initial conditions are chosen as $X(0) = [1 \quad 0]^T$ and $U(t) = 0$, for $t \leq 0$.

Random delay

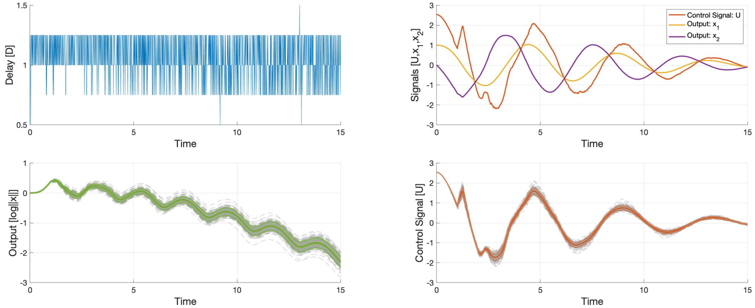
5 different delay values $(D_1, D_2, D_3, D_4, D_5) = (0.5, 0.75, 1, 1.25, 1.5)$ with transition probabilities satisfying the Kolmogorov equation

$$\begin{aligned} \frac{\partial P_{ij}(s,t)}{\partial t} &= -c_j(t) P_{ij}(s,t) + \sum_{k=1}^{r} P_{ik}(s,t) \nu_{kj}(t), s < t \\ P_{ij}(s,s) &= 1, \quad \forall i = j, \quad P_{ij}(s,s) = 0, \quad \forall i \neq j \end{aligned}$$

in which ν_{ij} and $c_j = \sum_{k=1}^{r} \nu_{jk}$ are positive-valued functions s. t. $\nu_{ii}(t) = 0$.

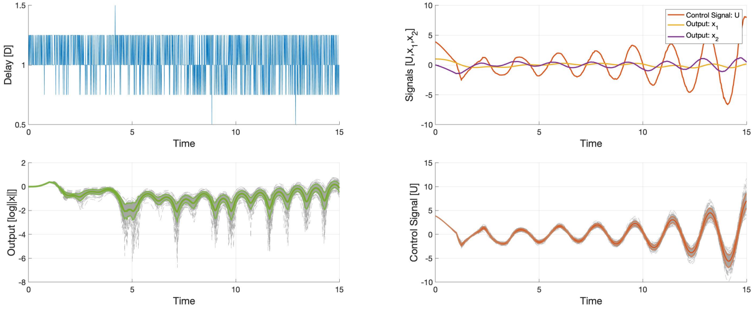
Toy example

$D_0 = 1$, Monte-Carlo simulations with 100 trials



Toy example

$D_0 = 1.25$, Monte-Carlo simulations with 100 trials



Outline

1. Robust compensation for linear systems
2. PDE transformations and Lyapunov analysis
3. Simulation results
4. Extension to nonlinear dynamics

Problem at stake

We consider the nonlinear plant

$$\dot{X}(t) = f(X(t), U(t - D(t)))$$

in which X an $\mathbb{R}^n$-valued random variable and $U \in \mathbb{R}$.

Delay definition

D is a Markov process with the following properties :

(1) $D(t) \in \{D_1, D_2, \ldots, D_r\}$, with $0 < \underline{D} \leq D_1 < D_2 < \ldots < D_r \leq \overline{D}$.

(2) The transition probabilities $P_{ij}(t_1, t_2)$, which quantify the probability to switch from D_i at time t_1 to D_j at time t_2 ($(i,j) \in \{1, \ldots, r\}^2$, $t_2 \geq t_1 \geq 0$), are differentiable functions $P_{ij} : \mathbb{R}^2 \rightarrow [0,1]$ satisfying

$$\sum_{j=1}^{r} P_{ij}(t_1, t_2) = 1, \quad (0 \leq t_1 \leq t_2)$$

Problem at stake

We consider the nonlinear plant

$$\dot{X}(t) = f(X(t), U(t - D(t)))$$

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Assumption 1

The dynamics $\dot{X} = f(X, U)$ with U scalar is strongly forward complete.

Problem at stake

We consider the nonlinear plant

$$\dot{X}(t) = f(X(t), U(t - D(t)))$$

in which X an $\mathbb{R}^n$-valued random variable and $U \in \mathbb{R}$.

Assumption 2

There exist a class $\mathcal{C}^1$ feedback law κ and a basin of attraction $\mathcal{A} \subset \mathbb{R}^n$ such that, for all $X(0) \in \mathcal{A}$, the plant $\dot{X} = f(X(t), \kappa(X(t)))$ is exponentially stable.

Prediction-based control law

$$U(t) = \kappa(P(t))$$

$$P(\theta) = X(t) + \int_{t-D_0}^{\theta} f(P(s), U(s))ds\,, \quad t - D_0 \leq \theta \leq t$$

Robust compensation

Theorem

For any compact $C \subset \mathcal{A}$, there exist positive constants ρ^* and $\varepsilon^\star(K)$ such that, if

$$|D_0 - D_j| \leq \varepsilon^\star(K), \quad j \in \{1, ..., r\} \quad \text{and} \quad \Upsilon(0) \leq \rho^*$$

there exist positive constants R and γ such that

$$\mathbb{E}_{[0,\Upsilon(0)]}[\Upsilon(t)] \leq R\Upsilon(0)e^{-\gamma t}$$

with

$$\Upsilon(t) = |X(t)|^2 + \int_{t-\overline{D}-D_0}^{t} U(s)^2 ds$$

<u>**NB :**</u> Assumption 2 is not restrictive in the sense that some asymptotically stable systems have zero delay-margin such as [L. Praly's textbook]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1(t-D) - x_2^3 \end{aligned}$$

For $D = 0$, the origin is asymptotically stable but is unstable for any $D > 0$.

Perspectives

This work is the subject of

- a submitted Automatica paper
- a second journal paper under preparation

Future works

- How to distinguish between delay distributions ?
- How to adapt the prediction horizon to the current delay distribution ?
- How to analyze the influence of the feedback gain and select it ?
- Extension to a continuum set of delays ?
- Methodology of stability analysis for cascaded PDEs or PDE-ODE.

Thank you for your attention !