A constructive algorithm for building mixing coupling potentials. Application to bilinear control.

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Despite the importance of control systems governed by a bilinear control for the description of phenomena that could not be modeled by additive controls, stabilization and controllability problems for such kind of systems have not been so widely studied in the literature as it happens for boundary and locally distributed controls. The main reasons of this fact might be found in the intrinsic nonlinear nature of such problems and furthermore, for controls that are scalar functions of time, in an ineluctable obstruction for exact controllability under very general assumptions presented in the celebrated work of Ball, Marsden and Slemrod [3].

By violating one of the hypotheses of [3], Beauchard and collaborators have succeeded in proving exact controllability for hyperbolic systems in a topology stronger than the natural one for such problems [4, 5]. Whereas, for evolution equations of parabolic type, results of rapid stabilization and exact controllability to a trajectory have been demonstrated by Alabau-Boussouira, Cannarsa and Urbani [1, 2]. A common step that can be found in all the aforementioned works is the proof of the exact controllability of the system obtained by linearization of the nonlinear problem along the reference trajectory. It turns out that a necessary condition on the rank of the potential must be satisfied. However, even if the genericity of such assumption has been proved for instance in [5], it is in practice difficult to exhibit examples of suitable potentials.

The aim of this talk is to present a constructive algorithm that provides a infinite class of functions that fulfill the required property.

References

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