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Title: Control of a third order in time dynamics governing nonlinear acoustic waves- a view from the boundary.

Abstract A third-order (in time) JMGT equation is a nonlinear (quasilinear) Partial Differential Equation (PDE) model introduced to describe a nonlinear propagation of high frequency acoustic waves. The basic model itself dates back to 19 century [works of Stokes], but modern applications have contributed new developments in terms of modeling and have triggered great interest in a contemporary applied mathematics. This has been instigated by a large array of applications arising in engineering and medical sciences-including high intensity focused ultrasound [HIFU] technologies, lithotripsy, welding and others. The important feature is that the model avoids the infinite speed of propagation paradox associated with a classical second order in time equations. Replacing a classical heat transfer by heat waves, gives rise to the third order in time derivative scaled by a small parameter $\tau > 0$, the latter represents the thermal relaxation time parameter and is intrinsic to the properties of the medium where the dynamics occurs. Other applications also include thermo-viscoelastic phenomenology.

The aim of the lecture is to provide a brief overview of recent results in this area which are pertinent to both linear and non-linear dynamics. From the mathematical point of view JMGT, can be seen in a variety of ways. A coupled dynamics between the second order hyperbolic system and an analytic semigroup. A more subtle interpretation is seeing JMGT as a *third order strictly hyperbolic system* , which however is non-symmetric and has a *characteristic* boundary. This feature has, of course, strong implications on boundary behavior [both regularity and controllability] which can not be patterned after classical hyperbolic systems theory [as it is the case for the wave equation]. One of the issue of interest is the so called "hidden regularity" -so fundamental to boundary control theory. The answer to this last question is subtle and delicate. It involves the analysis of strong Lopatinski condition which plays a major role in the theory of waves. As a consequence, the theory of regularity [both forward and inverse estimates] is particularly challenging- making the third order equation very interesting -even in the linear case.

Several recent results pertaining to boundary stabilization, optimal control and asymptotic analysis of the solutions with vanishing time relaxation parameter will be presented and discussed. In all these case, peculiar features associated with the third order dynamics leads to novel mathematical and phenomenological behaviors.