Controllability and stabilization for a degenerate wave equation in non divergence form with drift

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We consider the problem

$$\begin{cases} u_{tt} - a(x)u_{xx} - b(x)u_x = 0, & (t,x) \in Q, \\ u(t,0) = 0, t \in [0,+\infty), \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in (0,1), \end{cases} \tag{1}$$

where $Q = (0,+\infty) \times (0,1)$, $f \in L^2_{\text{loc}}[0,+\infty)$, $a, b \in C^0[0,1]$, $a > 0$ on $(0,1]$ and $a(0) = 0$. At $x = 1$ we consider different boundary conditions according to the considered problem. If we are interested in a controllability problem (see [2]) we assume

$$u(t,1) = f(t), \quad t \in [0,+\infty);$$

thus the function f acts as a boundary control and it is used to drive the solution to 0 at a given time T.

Otherwise, if we are interested in the stabilization problem (see [3]) we consider as a boundary condition the following damping one

$$y_t(t,1) + \eta y_x(t,1) + \beta y(t,1) = 0, \quad t \in [0,+\infty),$$

where η is a given function and β is a nonnegative constant. Clearly the presence of the drift term leads us to use different spaces with respect to the ones in [1] and it gives rise to some new difficulties. However, thanks to some suitable assumptions on the drift term, one can prove some estimates on the associated energy that are crucial to drive the solution to 0 at time T or to obtain a uniform exponential decay.

References

[1] F. Alabau-Boussouira and P. Cannarsa and G. Leugering, Control and stabilization of degenerate wave equations, *SIAM J. Control Optim.*, **55** (2017), pp. 2052-–2087.

[2] I. Boutaayamou, G. Fragnelli, D. Mugnai, *Boundary controllability for a degenerate wave equation in non divergence form with drift*, submitted.

[3] G. Fragnelli, D. Mugnai, *Linear stabilization for a degenerate wave equation in non divergence form with drift*, preprint.