Control of Parabolic Equations

F. Ammar-Khodja, A. Benabdallah, F. Boyer, M. González-Burgos,
M. Morancey, L. Ouaili, L. de Teresa

March 8, 2021

The aim of this talk is to give an overview on controllability of parabolic equations. Of course, my objective is more restrictive: I will consider only **null controllability of linear and parabolic equations**:

$$\begin{cases} y' = Ly + Bu \\ y(0) = y^0 \end{cases}$$

By parabolic equations, we mean:

1. H, U Hilbert spaces
2. $(L(D(L))$ generator of a $\mathcal{C}_0$ semigroup on H
3. $\sigma(L^*) = \Lambda = \{\lambda_k\}_{k\geq 1} \subset \mathbb{C}$ satisfying

$$\begin{cases} \lambda_i \neq \lambda_k, \quad \forall i, k \in \mathbb{N} \text{ with } i \neq k, \\ \Re(\lambda_k) \geq \delta |\lambda_k| > 0, \quad \forall k \geq 1, \ \Re(\lambda_k) \underset{k\to+\infty}{\to} +\infty. \end{cases}$$

4. $\Phi := \{\phi_k\}_{k\geq 1}$, eigenvectors of L^* associated to Λ is a complete family of H
5. B is an admissible control operator for the semigroup generated by $(L, D(L))$, i.e, $B \in \mathcal{L}(U, D(L^*)')$ and

$$\mathrm{R}(L_T) \subset H, \quad L_T u = \int_0^t e^{(T-s)L} Bu(s)\, ds, \quad u \in L^2((0,T); U).$$

Our goal will be to analyze the application

$$(\Lambda, \Phi, B^*) \mapsto T_{min}$$

where $T_{min} \in [0, +\infty]$ is defined by

$$\forall T > T_{min}, (L, B) \text{ is null controllable at time } T,$$

$$\forall T < T_{min}, (L, B) \text{ is } \textbf{not} \text{ null controllable at time } T.$$